Minimum Redundancy Linear Arrays

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The idea of using minimum redundancy linear arrays for radio interferometers has been familiar to radio astronomers since Al Moffett wrote his well known IEEE paper on the subject in 1968(AP-16, 172). For the classic East-West linear arrays, the minimum redundancy array provides essentially an exact optimum solution for the choice of antenna separations for a 12 hour earth rotation synthesis.

For two dimensional arrays, there is no corresponding exact solution, and the usual procedure is to use a trial and error method (Monte-Carlo is a fancier term) or some non- linear relaxation optimization scheme such as simulated annealing. The latter is very compute intensive, but it is a good way to proceed to an optimum once you have a rough idea of what will work. Especially for arrays of a small number of elements, minimum redundancy is important. It provides the maximum ratio between the angular resolution of the array and the largest size structure that can be mapped (that is not filtered out) with no gaps in sampling sizes in between these two limits. The mapping is well defined. Although linear minimum redundant arrays do not provide an exact solution for two dimensional arrays, they are a useful starting point in the design. The initial strategy for the design of all the BIMA arrays has been the use of linear minimum redundancy arrays along the EW and NS arms of the T shaped track.

The problem of minimum redundancy arrays was studied originally by a mathematician, Leech (J. London Math Society, 1956, 31, 160). Leech studied arrays that contained all multiples of a unit spacing, $a(i.e. 1a, 2a, 3a, ...Na)$ up to some maximum spacing N. Leech showed that a completely non-redundant linear array is possible only up to 4 elements; the spacings for a 4-element array are $*1*3*2*$, and the array includes all multiples of a from $1a$ to $6a$. Thus, there are six equal spacings with no repeats, and that is just the number of combinations of 4 elements taken two at a time.

For 5 or more elements, it is no longer possible to build a non-redundant array that includes all the multiples of a. Instead, there are two possibilities: (1) Include all the multiples of a unit spacing up to some maximum, and then allow a redundancy for some of those multiples. This is called a restricted minimum redundancy array; for 5 elements, the two possible restricted arrays have element spacings of $*1*3*3*2*$ and $*1*1*4*3*$, both of which include all

spacings from $1a$ to $9a$. The former includes a redundancy at $3a$ and the second includes a redundancy at $1a$. The maximum number of independent spacings is 9, whereas the number of combinations of 5 elements taken 2 at a time is 10. (2) Include all the multiples of a unit spacing up to some intermediate spacing, then allow a gap where some multiples of a are not included. For 5 elements, these unrestricted arrays are $*3*1*5*2*$, which includes all spacings to $9a$, then there is a jump to $11a$; and $*4*1*2*6*$, which includes all spacings to $9a$, then there is a jump to $13a$.

For our application, since gaps in the uv plane cause sidelobe and sampling problems, the restricted minimum redundancy arrays are the better choice. Note that the redundancy cannot be broken while keeping uniform sampling – the emphasis really is that this is a $*[minimum]*$ redundancy array. If one tries to push antennas around and use, say, $*1*2.7*3.3*2*$ instead of $*1*3*3*2*$, then some of the sampling is squeezed too tight and also some gaps are introduced. The minimum redundancy arrays are the arrays that cover the line uniformly, with the maximum number of spacings for a given number of elements and no gaps.

For 6 antennas, there are 3 restricted linear arrays: $*1*1*4*4*3*$, $*1*5*3*2*2*$, and $*1*3*1*6*2*$. Each of these gives 13 independent spacings along a line, whereas there are 15 combinations of 6 elements taken 2 at a time. Results for larger arrays may be found in Leech. In general, the redundancy is typically about 15%.

1 Application to a 2-dimensional Array

There may be many strategies for designing 2-dimensional arrays. For the purpose of this memo (which describes array choices for the short-term only), we have restricted our study to arrays that lie at positions on the T. We could almost certainly do better if we allowed true freedom in 2 dimensions, and this issue should be studied thoroughly before we consider pouring new deep stations on or off the T.

Since we're restricted to the T, and since we have a small number of antennas, it is reasonable to start with Leech's scheme for designing an array. There are two separate advantages to this choice: it produces uniform and complete sampling in the EW and NS tracks, and it also produces relatively uniform sampling among the diagonal tracks. If the elements are approximately evenly divided between the EW and NS legs, the number of diagonals tracks will be maximized and will be half of the total number.

1.1 The Most Compact Array

We begin with a minimum redundancy NS array connected to the middle of a minimum redundancy EW array at its middle, with approximately equal num-

bers of elements on each leg.

1.1.1 The Central part

A zero spacing is, of course, not possible, and we put three antennas as close to one another as possible in a triangle in the center. The antenna diameters are all 20 feet, so we put one at 20 north, one at 15 east, and one at 15 west. This gives 25 foot spacings on the diagonals and 30 feet EW. Collisions are possible, and care must be taken for protection. With baseline foreshortening during the tracking, this arrangement gives good uv coverage all around the edge of the 20 foot radius hole in the center of the uv plane defined by the minimum unshadowed spacing.

1.1.2 The Unit Spacing

With uniformly illuminated square aperture antennas, the unit spacing required for Nyquist sampling would be just one antenna diameter. For round antennas with tapered illumination, the unit spacing should be about 0.8 of an antenna diameter. On the other hand, during most of the scan there is foreshortening of the spacings. Thus, we adopt 20 feet as the approximate unit spacing.

1.1.3 The North-South Arm

For the North-Sourth arm, we select the five antenna minimum redundant group $*1*3*3*2*$, with the unit spacing adjacent to the T intersection. A 20 foot antenna separation is actually too close. We use the 15E and 15W antennas as an effective single antenna at the T intersection and the first 20N station gives us the approximate 1 unit first spacing. With a 20 foot unit, the next stations in the above pattern are at 80N, 140N, and 180N.

1.1.4 The East-West Arm

At this point we have 4 antennas on the NS arm and antennas at 15E and 15W. Altogether, we end up with 6 antennas on the EW arm, with a 30 foot spacing at the T intersection (approximately a 20 foot unit spacing). A 6 element minimum redundant group with a unit spacing in the center may be found in Leech: $*1*3*1*6*2*$. This does not have perfect East-West symmetry with respect to the center of the T. We make a few small adjustments in the spacings by trail and error and finish with the following set of East-West stations: 100W, 75W, 15W, 15E, 125E, 165E. Reading from West to East, one gets close to the $*1*3*1*6*2*$ pattern with a 20 foot unit separation, a little stretched on the West and a little squeezed on the East.

1.2 Summary

This compact (C) array has stations at: 100W,75W,15W,15E,125E,165E,20N,80N, 140N, and 180N.

Because minimum redundant linear arrays are used on both the NS and EW arms, the tracks for the separate arms have the maximum number of tracks, uniformly spaced, out to the longest baseline in units of one antenna diameter. The diagonals are also non redundant and fairly uniformly separated. The net result is that the individual uv tracks close on one another with a total scan of about 8 hours, for all declinations. Unlike the classic East-West arrays, which require a full 12 hour scan, this two dimensional array needs just 8 hours. The total extent along the NS arm is 9 antenna diameters; the EW extent is 13. The maximum spacings NS are actually furnished by the diagonals and average 11 antenna diameters. This configuration has approximately circular beams at all declinations.

An inspection of the uv tracks shows fairly uniform uv coverage with few gaps larger than 20 feet, except for declinations near the equator and the 40 foot diameter hole in the center. The dynamic range in map size scales is about 10. For example, at 100 Ghz the average resolution is about $7.5''$, and the maximum source size that can be adequately sampled is about $70'' - 75''$ in extent.

It turns out that with two dimensional arrays that have minimum redundancy the dynamic range in map scales is about equal to the number of array elements. This is in contrast with East-West linear arrays for which the dynamic range is proportional to the square of the number of elements. For sources close to the pole where such arrays provide round beams, this comparison favors the East-West arrays. However, these arrays give very elliptical beams at lower declinations and also require a 12 hour synthesis. The two dimensional arrays are therefore more generally useful.