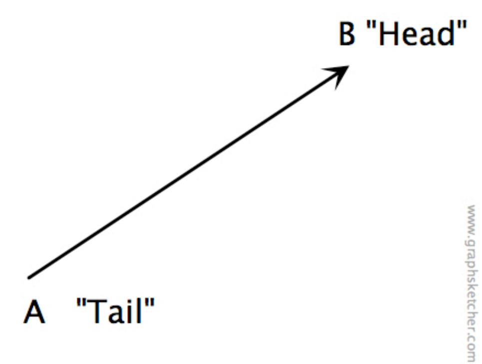
Lecture 1: Introductory Topics

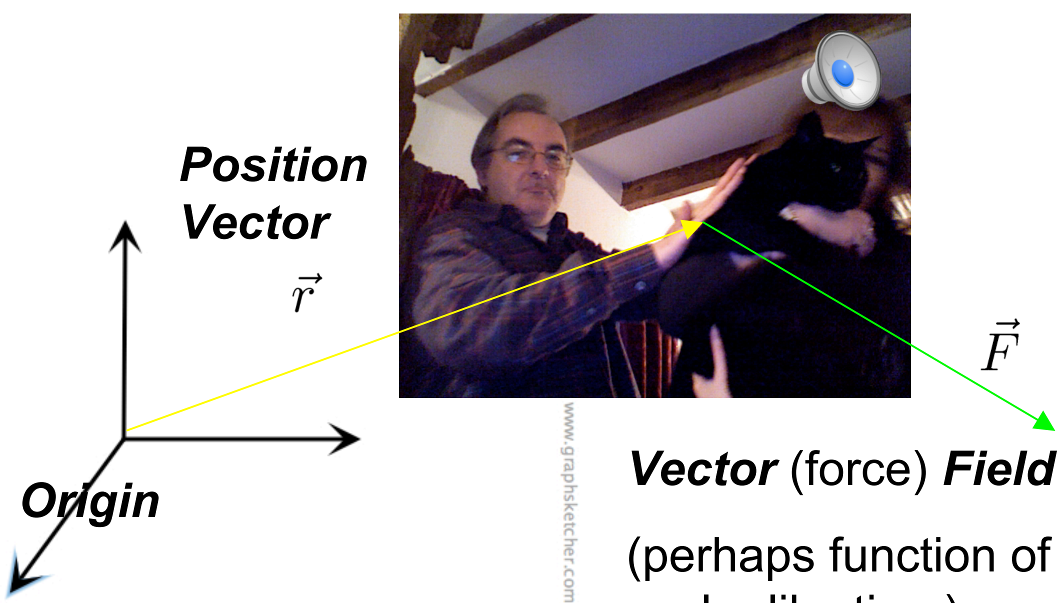
- Intuitive “Physicist’s” approach to ***Vectors***
- Normally 3D, sometimes 2D (c.f. complex #s)
- Have ***Magnitude*** and ***Direction*** but not ***Position***

- Localized force requires two vectors



$$\vec{V} = \vec{AB} = (V_1, V_2, V_3)$$

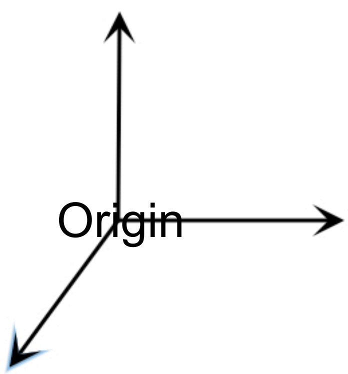
Where V_1, V_2 and V_3 are components w.r.t. $[\vec{i}, \vec{j}, \vec{k}]$ basis set



Vector (force) ***Field***

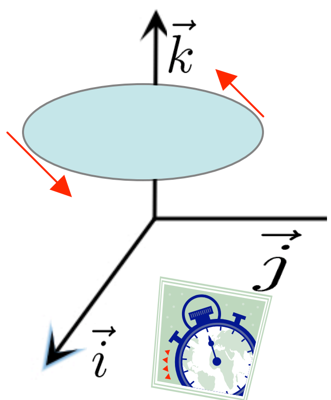
(perhaps function of a scalar like time)

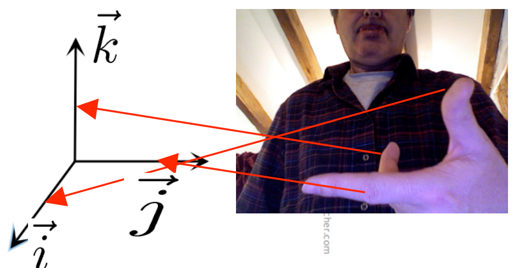
Translation



- ***Parallel transport*** of a vector; vectors are “moveable”
- Vectors are said to be ***invariant under translation*** of coordinate axes

Rotation

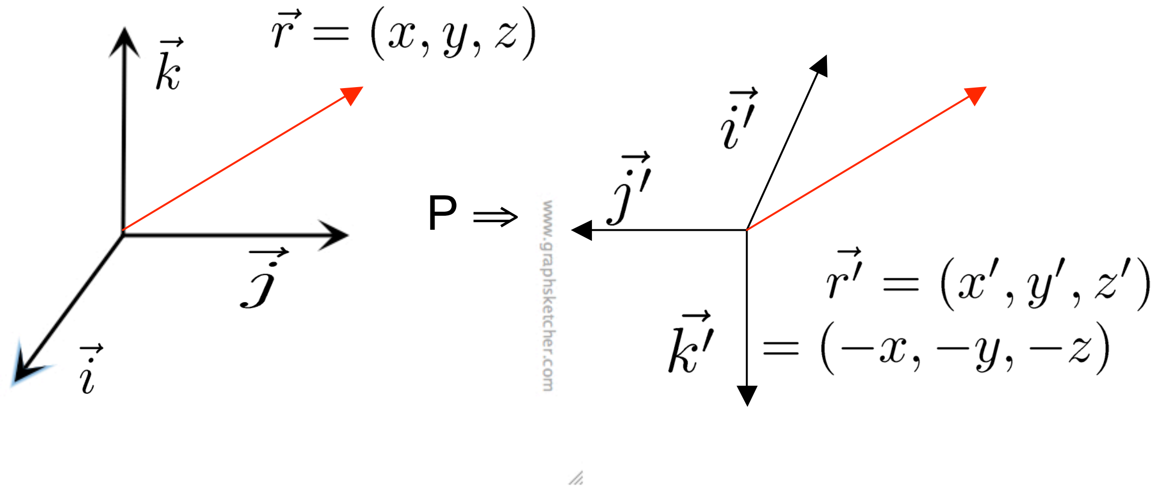


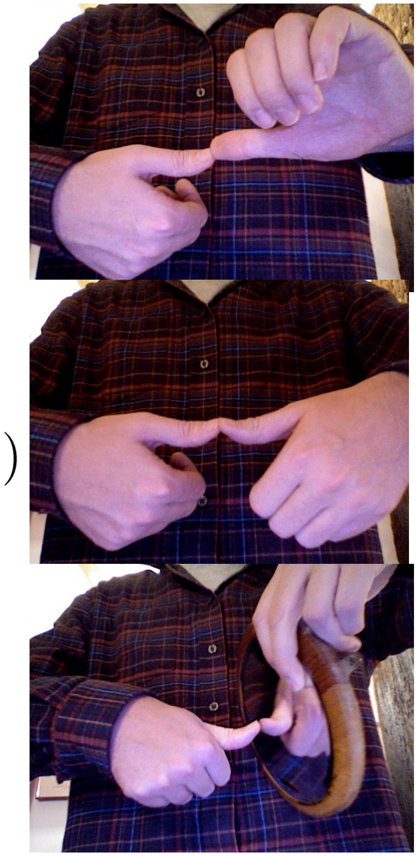


RIGHT-HAND RULE !

- The magnitude (but obviously not direction) of a vector is ***invariant under rotation*** of coordinate axes
- Convention: "Right-hand corkscrew rule", positive rotation appears clockwise when viewed parallel (rather than anti-parallel) to the vector; or "Tossed clocks are left-handed"

Inversion

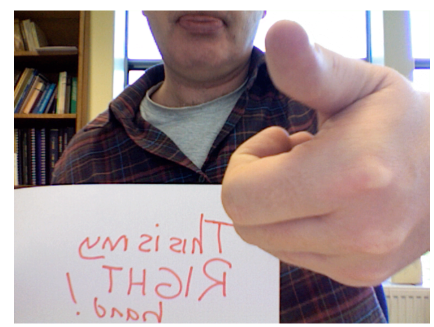




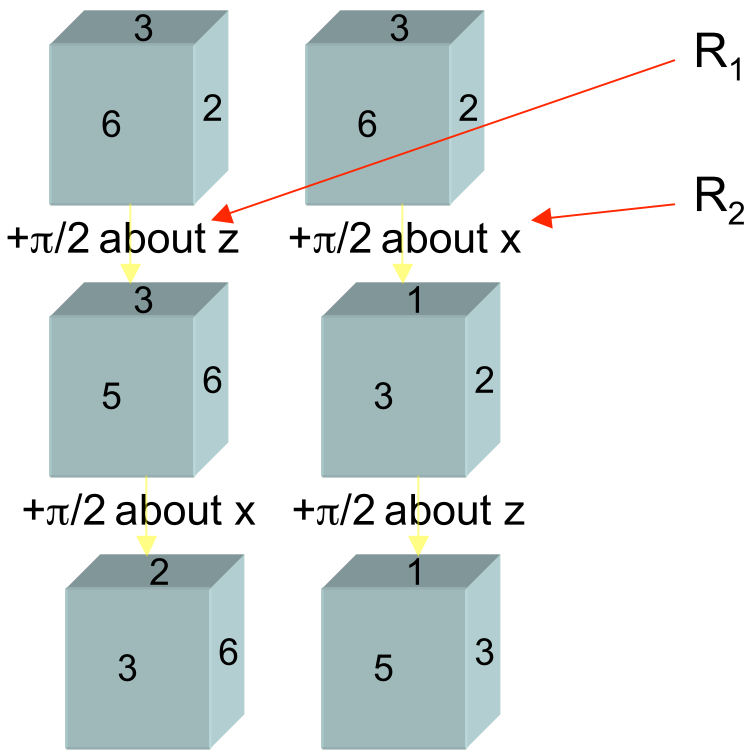
P

PR

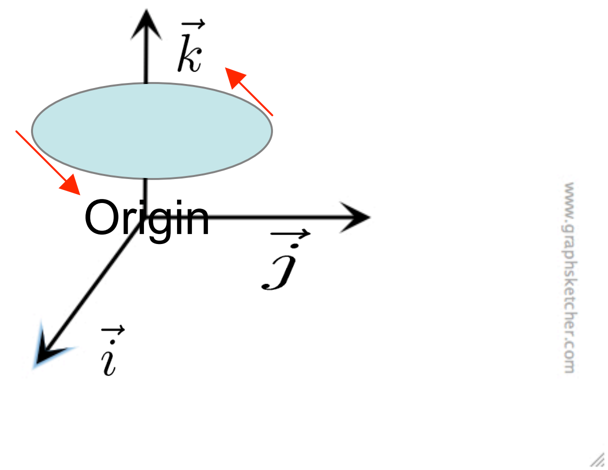
- ***Inversion*** of coordinate axes (the ***parity P*** transformation) is a reflection of co-ordinate axes in origin
- Right-handed set $[\vec{i}, \vec{j}, \vec{k}]$ becomes left-handed set $[\vec{i'}, \vec{j'}, \vec{k'}]$
- ("Proper") or ***Polar*** vectors are ***odd***, i.e. they reverse in sign under inversion of axes $\vec{A}(-\vec{r}) = -\vec{A}(\vec{r})$
- Reflections in a plane are equivalent to PR=RP where R is a rotation
- Any position vector is an example of a polar vector



Intuitive approach sometimes hazardous



$$R_1 \times R_2 \neq R_2 \times R_1$$



- Rotation not in general ***commutative***
- Breaks one of the "hidden rules" of ***vector spaces***
- Rotation through a finite angle is not a vector

Or with Earth

+π/2

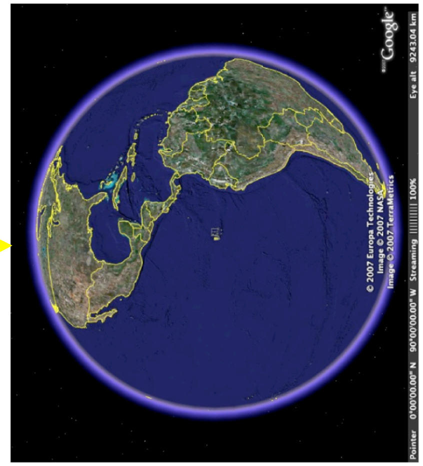
about

z

+π/2

about

x



+π/2

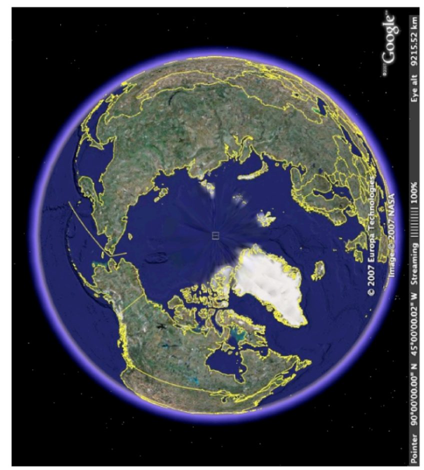
about

x

+π/2

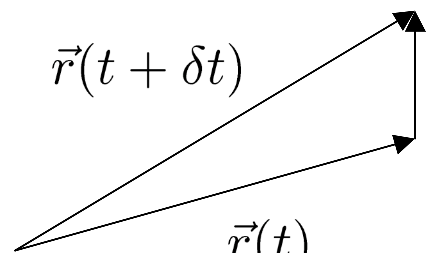
about

z



Differentiation and integration of vectors

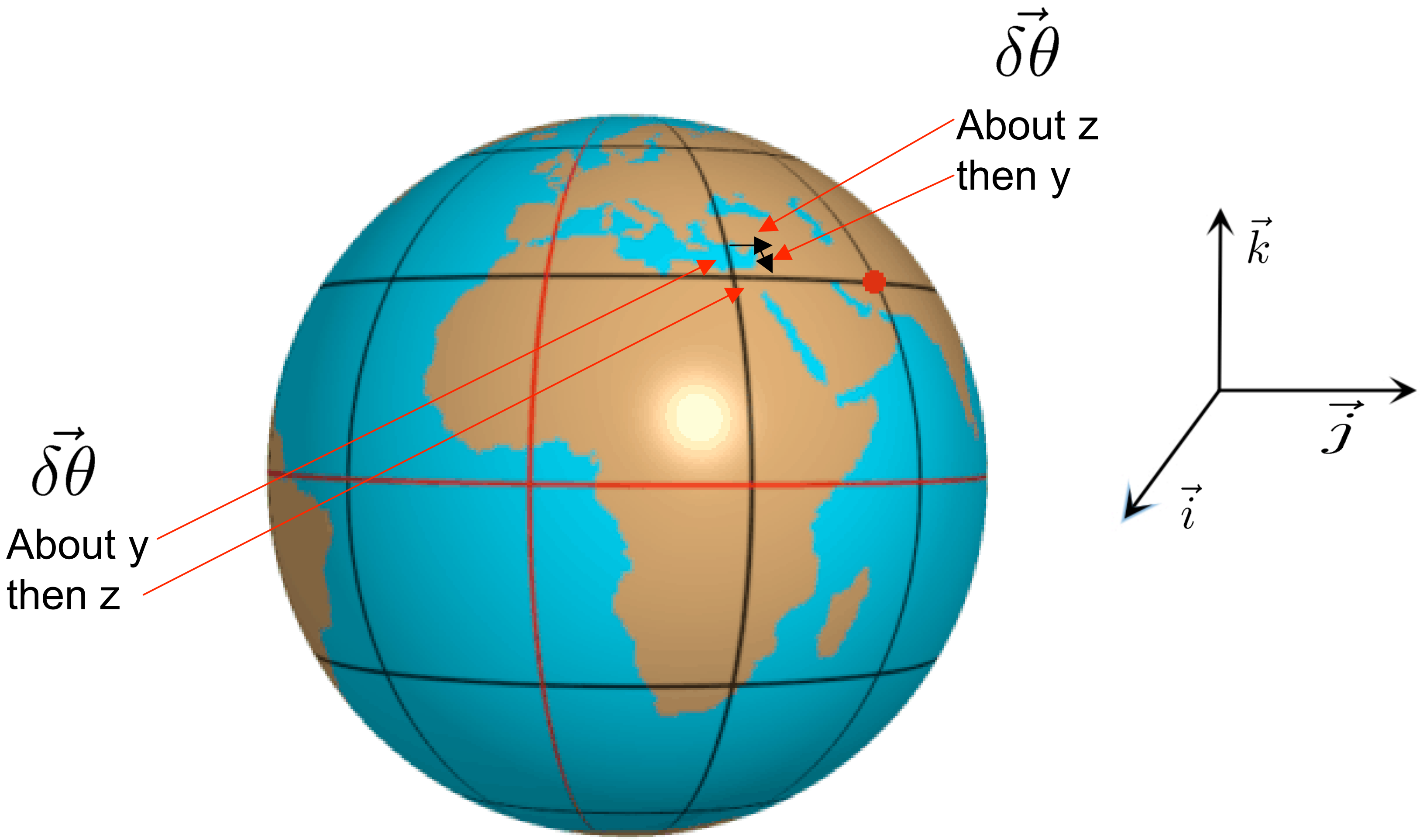
$$\frac{\mathrm{d}\vec{r}(t)}{\mathrm{d}t} = \lim_{\delta t \to 0} \frac{\vec{r}(t+\delta t) - \vec{r}}{\delta t}$$



- Integration w.r.t. scalar is inverse operation remembering that the integral, and constant of integration, has same nature (vector) as the integrand
- From polar vector $\vec{r}(t)$ we get a family of polar vectors

$$\text{Velocity } \dot{\vec{r}}(t) \text{ and Momentum } \vec{p} = m\vec{v} = m\dot{\vec{r}}$$
$$\text{Acceleration } \vec{a} = \dot{v} = \ddot{\vec{r}} \text{ and Force} = m\ddot{\vec{r}}$$
$$\text{Electric Field } \vec{E} = \vec{F}_{\text{electrostatic}}/q$$

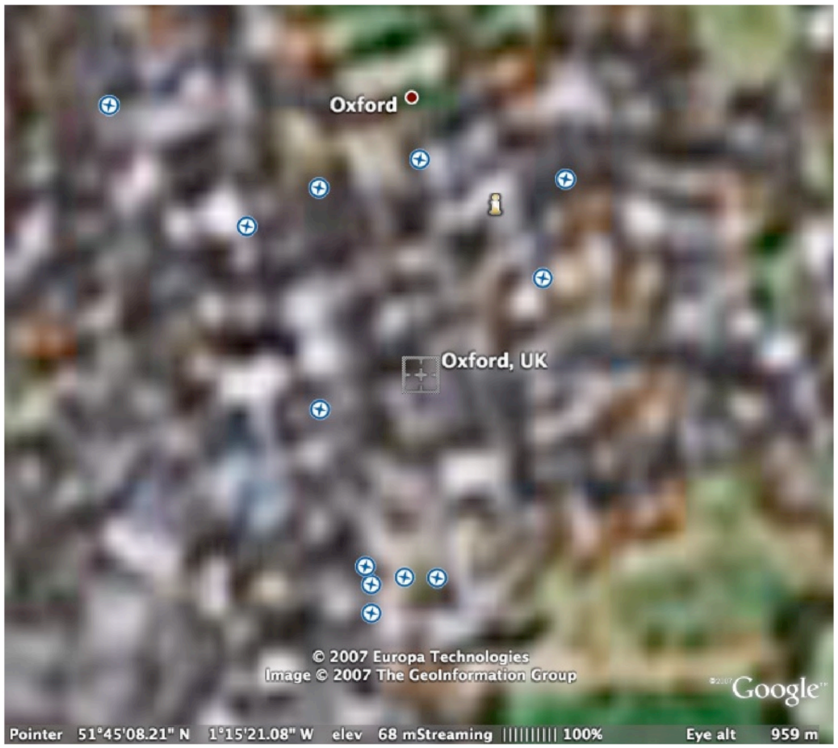
Small rotations as vectors



IFF angles small, end up at the same place, so $\delta\vec{\theta}$ is a vector as is

angular velocity $\vec{\omega} = \frac{\mathrm{d}\theta}{\mathrm{d}t}$

Or with Earth



Rotating Earth from Oxford to Cambridge: ignoring the non-sphericity of the Earth, we'd still end up on Parker's Piece regardless of the order of the rotations!

Pseudovectors

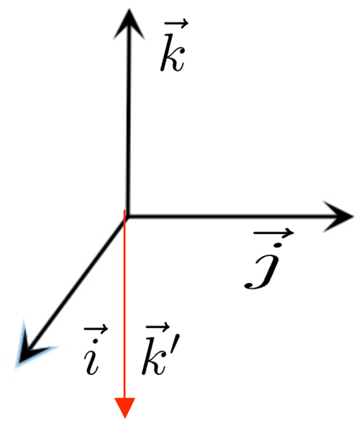
- Parity transformation (inversion of coordinate axes) leaves $\vec{\omega}$ unchanged
- $\vec{\omega}$ is ***even:*** $\vec{\omega}(-\vec{r}) = \vec{\omega}(\vec{r})$
- Example of an ***axial vector*** or ***pseudovector***
- Note that any reflection operation (e.g. in xy plane) changes from a right-handed set to a left-handed set
- Cross-product generates axial from polar vectors:

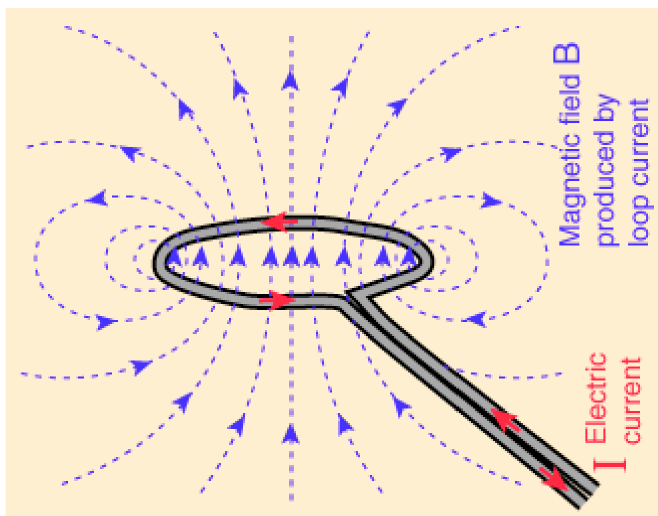
$$\text{Torque } \vec{G} = \vec{r} \times \vec{F} \text{ and Angular Momentum } \vec{L} = \vec{r} \times \vec{p}$$

- [Polar] = [Axial] x [Polar]:

$$\vec{v} = \vec{\omega} \times \vec{r} \text{ and } \mathrm{d}\vec{F} = I\mathrm{d}\vec{r} \times \vec{B}$$

- Hence torque, angular momentum,angular velocity and magnetic field are all axial vectors





Vector areas

- Define ***scalar area*** $|\vec{S}| = S$
- Direction perpendicular to S with a RH rule to assign unique direction
- Component of $\vec{S}$ in a direction is projected area seen in that direction
- Area is a ***pseudovector***
- Break into many small joined planes $\vec{S}_{\mathrm{resultant}} = \lim_{n\to\infty} \sum_n \vec{S}_{\mathrm{smallplanes}}$

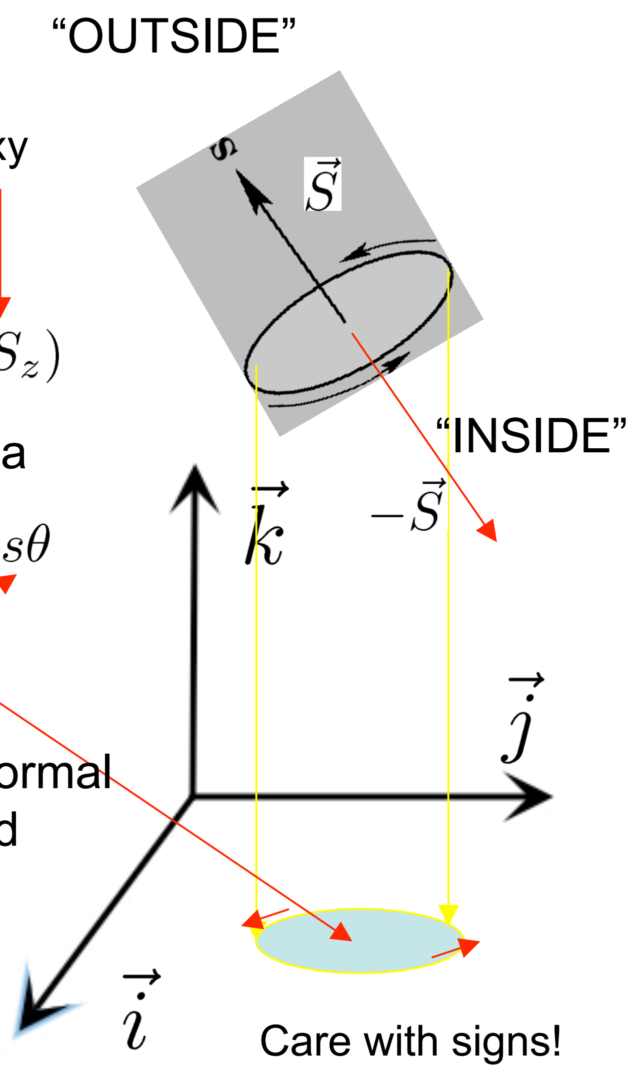
Project onto yz,xz,xy planes

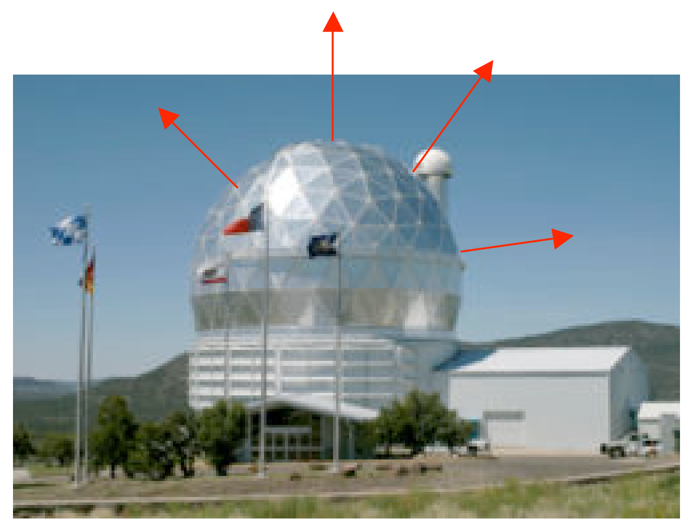
$$\vec{S} = (S_x, S_y, S_z)$$

Projected area

$$S_z = |\vec{S}| cos\theta$$

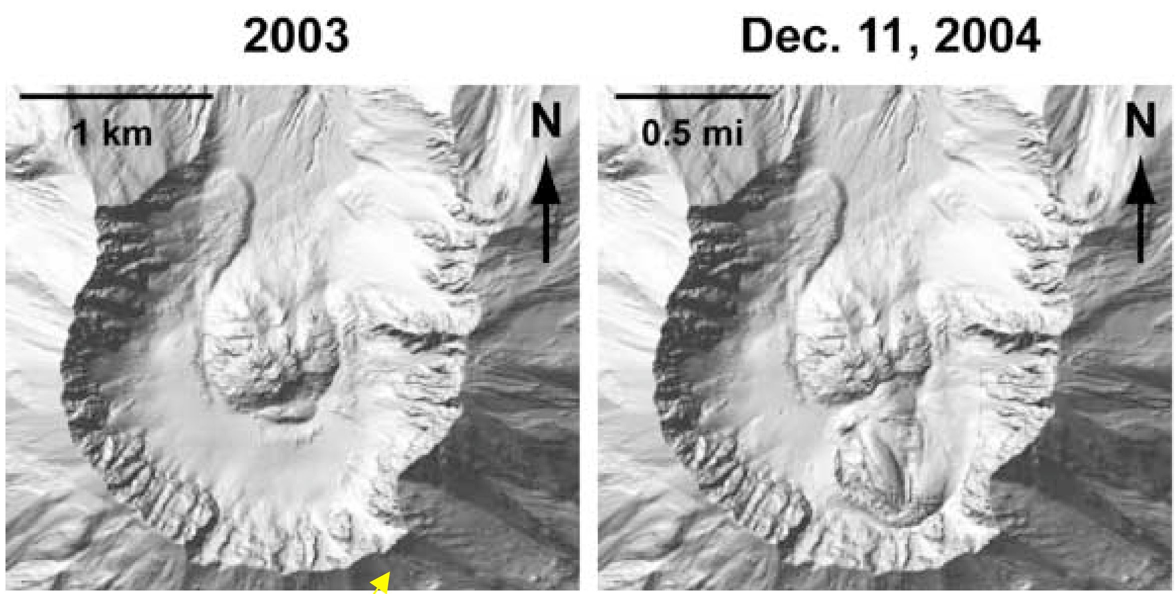
Angle between normal to the surface and vector (0,0,1)

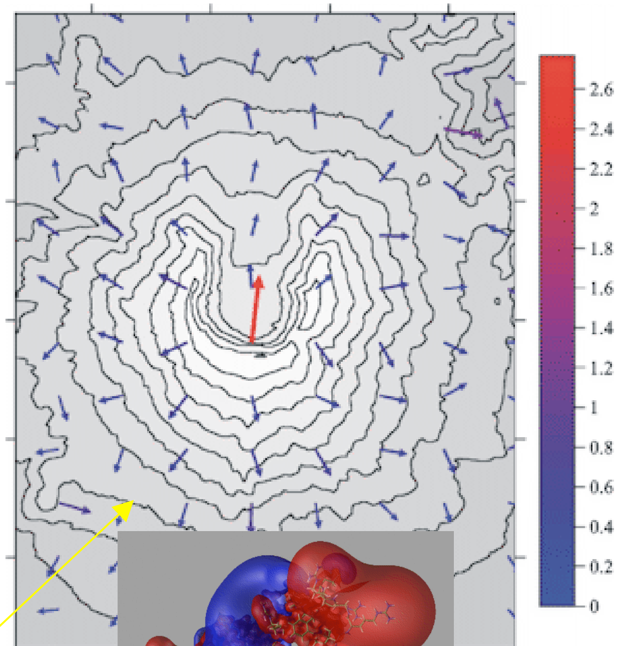


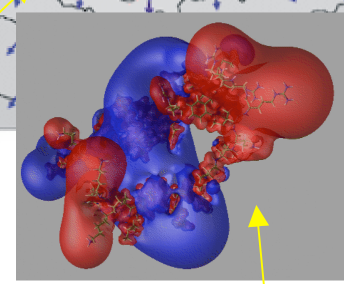


Or something much more crinkled

Scalar Fields





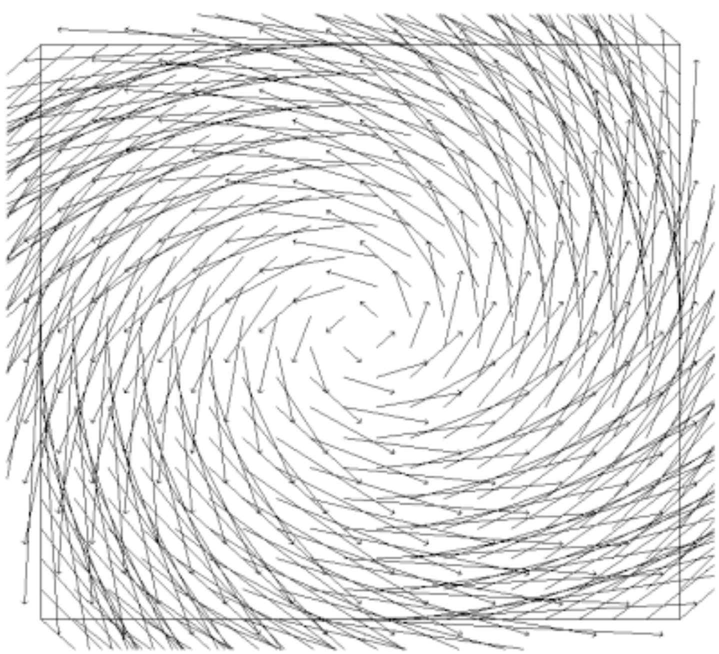


• Assigns a *scalar* (single number) to each point, possibly as a function of other scalars like time: e.g. 2D function like height $h(x, y, t)$

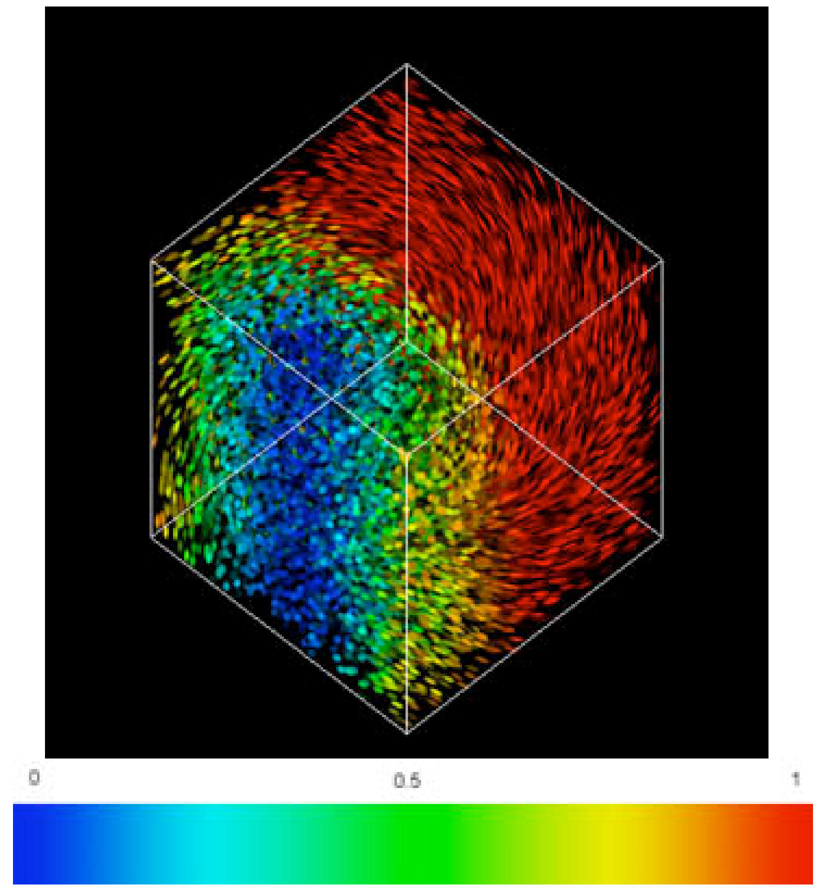
• For 2D scalar fields chose between using 3rd dimension to represent value (e.g. ***relief map*** for height) of function or ***contour map*** (join points of constant h).

• In 3D, $\phi(x, y, z, t) = \phi(\vec{r}, t)$ with ϕ temperature or other scalar; ***contour surfaces***

Vector Fields



2D $\vec{A}(\vec{r}) = (-y, +x)$



- Assigns a single vector (in 2D, 2 numbers; in 3D, 3 numbers) to each point, possibly as functions of other scalars like time $\vec{A}(x, y, z, t) = \vec{A}(\vec{r}, t)$
- Examples: velocity field of a fluid, electric and magnetic fields