MINT Optics

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May 2, 2000

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This paper discusses the design and the preliminary beam test on the optical system for the MINT experiment. A set of optimized antenna parameters was chosen for a shielded Cassegrain system using antenna pattern code *DADRA*. The design and drawing of the physical layout of the antenna were sent to the machine shop to be made, keeping the cost low. The accuracy of the parabola of the primary mirror was verified up to 1 mil. A jig was made to mount and center the secondary mirror to an accuracy of 2 mil. Near-field beam mapping of the horn and antenna was done using a modulated source. The results were compared to antenna pattern obtain from *DADRA*.

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1 Introduction

The characterization of the angular power spectrum of the cosmic microwave background (CMB) anisotropy will help distinguish the various comological models. A summary of our theoretical knowledge is given in Hu 2000 [1], while Page & Wilkinson 1999 [2] give the observational summary.

Recently, with the advances in low-noise, broadband, millimeter-wave amplifiers (100 GHz and up), interferometers are emerging as useful instruments for probing the CMB anisotropy from the ground.[2] Since an interferometer directly measures the Fourier transform of the sky signal, it has the advantage over single-dish experiments of determining directly the CMB angular power spectrum -- the goal of most CMB anisotropy experiments – with well-defined window functions.[3] In addition, interferometers are less susceptible to detector noise, ground signal pick-up, and atmospheric signal corruption associated with single receivers because only signals common to pairs of receivers are correlated for detection. Finally, interferometers allow higher angular resolution without the building of large antennae, for it is the baseline[4] and not the dish diameter that sets the angular scale (for a given wavelength) of the instrument. This short paper will describe the design and the preliminary test of optical system used in the Princeton Microwave INTerferometer (MINT) experiment.

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2 MINT Design

MINT is a dedicated four-element CMB interferometer that operates at around 145 GHz ($\lambda \sim 2$ mm) with a bandwidth of 4 GHz. The longest baseline is about 1 m, and the shortest about 32 cm. Hence, MINT will probe angular scales of 5 to 15 arc minutes (l = 1000 to 3000). The details of feed arrangement on the *uv* plane to give equal l-space coverage were worked out by Hinderks[4] and Dieguez [5].

The MINT detectors are identical to those used in the *D*-band (144 GHz) channels used in the MAT experiment [6]. MINT uses SIS mixers to convert radio frequency (RF) signals to an intermediate frequency (IF) band of 4 to 6 GHz, where they are then amplified by HEMT amplifiers. The amplified IF signals are then mixed down to four bands at 0 to 500 MHz, where they are digitized and correlated. Details of the MINT electronics are outlined in [7] and [8].

For rapid measurement of the CMB power spectrum, MINT needs compact antennae for close packing to give high sensitivity. On the other hand, the antennae have to be low scattering to minimize inter-antenna coupling. As a compromise between compactness and low scattering, MINT will use shielded Cassegrain antennae in a planar array.

[2] There are at least four other CMB interferometers planned for observations in the next two years: VSA (Cambridge University), DASI (U of Chicago), CBI (Caltech) and Tenerife (Jodrell Bank).
[3] Details relating an interferometer response to the CMB power spectrum is outlined in White, 1997 [3].
[4] The distance between two antennae in an interferometer is called the baseline.

3 Optics and Antenna Parameters

Each MINT antenna consists of a classical Cassegrain optics with a 5° conical shield that guides radiation from secondary scattering to the sky. The top edge of the cone is rolled with radius of a few wavelengths to reduce diffraction from the edge of the shield itself. (See Figure 1a)

Cassegrain System: The usual parabolic reflector antenna, with a feed at the focus, does not allow much control over the aperture[5] power distribution except for what is achievable by changing the focal length of the parabola. The Cassegrain system, consisting of two reflecting surfaces -- a concave parabolic main dish and a convex hyperbolic secondary – has an extra degree of freedom to control the aperture field distribution. One could, for example, reshape both the secondary and the main reflector to change the power distribution on the antenna, but still maintain the required phase distribution [9] [6].

In general, Cassegrain antennae have shorter main reflector focal lengths, and hence are more compact than conventional parabolic reflectors, but suffer performance degradation due to substantial interference from the secondary mirror. Additional benefits of Cassegrain system include the ability to place the feed at a convenient location, and to reduce spillover and side-lobe radiation. Usually, the size of the secondary must be at least a few wavelengths in diameter (d) to serve as an efficient reflector. However, it must be small enough to reduce "shadowing" that degrades the gain of the antenna. Thus, the main reflector or primary is usually large compared to the secondary ($D >> d$, usually D is greater than 50λ), with antenna gain[7] of 40 dB or greater.

Equivalent parabola: One can understand and relate the performance of Cassegrain antennae to that of a single-parabolic reflectors by using the concept of *equivalent parabola*. The composite system of primary and secondary mirrors is now replaced by an equivalent focusing surface (shown as a dashed line in Figure 1b). Using simple ray tracing or geometric optics, the equivalent focusing surface is just a paraboloid with *equivalent* focal length, F. The distance, F, is measured from the paraboloid vertex and the real focal point (where the feed resides). The boundary of this paraboloid is defined by the intersection between incoming rays parallel to the antenna axis and the extension of diverging rays from the real focal point. F is generally longer[8] than f, the focal length of the primary. A parabolic reflector with a long focal length has less taper in its aperture field distribution and has better scanning performance (less loss in gain as the feed is moved off axis) [17]. Hence, a Cassegrain system, being equivalent to a parabolic reflector of longer focal length, has the advantage of being compact (shorter f), while maintaining the RF performance of a system with longer F. In other words, the Cassegrain arrangement of a secondary over a primary "magnifies" the parabolic primary[9].

Antenna Parameters: The Cassegrain geometry is described completely by four independent parameters [12]. The following four parameters are chosen to define the Cassegrain system (See Figure 1c):

Main reflector or Primary diameter, D
Primary focal length, f
Distance of feed behind primary or Back focal distance, z
Half-angle subtended by secondary, θ_s

The first two deal with the primary reflector; the second two deal with the secondary. Other parameters like the eccentricity of the secondary, the magnification of the system, etc. can be derived from the above using formulae given in Appendix A.

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[5] The aperture is defined as the plane at the real focal point (where the feed is located) perpendicular to the antenna axis. It is known as the focal plane.
[6] The paper by Haeger and Lee [10] that compared shaped and nonshaped small Cassegrain antenna could be relevant to our case if we decide to further fine tune our optics.
[7] Here, "gain" is actually the maximum gain of the beam or the directivity D_{max}. Optical definitions and conventions in this paper follow those outlined in L. Page 1998 [11] and are explained in Appendix B.
[8] This is true so long as the primary is concave; the secondary convex.
[9] $F = Mf$, where M is known as the magnification. See Appendix A for the formula relating M to e, the eccentricity of the hyperbolic secondary.

MINT will explore the CMB anisotropy up to a maximum angular scale or minimum angular momentum harmonics, *l*, of about 1000. This requires a minimum baseline of 32 cm. For maximum sensitivity, we chose primary diameter, D = 30 cm, the maximum size dish possible.[10]

The remaining three parameters are determined based on various RF considerations, beam aberrations and antenna coupling using *Diffraction Analysis of a Dual Reflector Antenna (DADRA)*. For given main and sub-reflector surface shapes and boundary geometry, *DADRA* uses Physical Optics (PO) of equivalent surface currents to calculate both far-field and near-field antenna patterns and gains. It utilizes a triangular facet representation of the reflector surface. Details regarding computation routines and procedures are documented in [13]. We iterated various input antenna parameters in *DADRA* to arrive at an antenna beam of desirable features. Details of the optimization of antenna parameters are outlined in Loh [14].

Figure 2 and 3 gives the final beam pattern and current distributions from *DADRA*, with various parameters and beam characteristics tabulated in Table 1.

Table 1: Parameters of Cassegrain Antenna, Beam and Current Characteristics

Parameter	Symbol	Value
Primary diameter	D	30 cm
Primary focal length	F	12 cm
Back focal distance	z	1.0 cm below primary vertex
Half-angle subtended by secondary	θ_s	23.0^O
Primary focal ratio or "speed"	f/D	0.4
Half-angle subtended by primary	θ_p	64.01^O
Secondary eccentricity	e	1.965
Secondary diameter	d	9.144 cm
Secondary directrix	p	4.8171
Secondary interfocal length	F_s	13 cm
Exit focal ratio	f_s/d	1.422
Magnification	M	3.073
Equivalent focal length	F	36.87
Equivalent focal ratio	F/D	1.229
Maximum gain or Directivity	D_{max}	48.4 dB
Full width at half maximum	$FWHM$	0.484^O
Current edge taper on primary	y_e^{pri}	20.2 dB below maximum
Current edge taper on secondary	y_e^{sec}	13.1 dB below maximum

4 Physical Design and Secondary Mount

For ease in calibration and storage, the Cassegrain antenna system is separated to three modules: primary dish, secondary and mount, and ground-shield (Figure 4).

Primary dish: Simple machine code that takes into account the size of the cutting tool was written to allow the parabolic surface to be cut up to an accuracy of 1 mil = 0.025 mm, or 1/78 λ. The machined surface was verified to follow a parabola contour using a dial indicator (Figure 5). For light weighting and structural strength, the back of the mirror was hollowed, leaving only thin ribs of honey combed shape (Figure 6).

Secondary and mount: Like the primary, the hyperbolic surface of the secondary was cut to an accuracy of 1 mil. A mounting ring tapered at 5 degree (to match the shield) was designed to allow secondary and its three G10 support posts be detached easily from the primary dish. The mounting of the secondary with respect to the primary parabola need to be good to 1 mm for the beam not to degrade [14]. Figure 7 gives the numerically computed beam (from *DADRA*) when the secondary is displaced laterally by 1 mm. A dedicated jig was made (Figure 8) to place and center the secondary at the correct position to a circumference accuracy of 2 mil using a dial indicator.

[10] We are constrained to by the need to build flanges for bolt holes around the primary dish for mounting purposes.

Random Phase Error: One way to model the surface roughness of a reflective surface is by a random Gaussian distribution. This gives the Ruze formula [9] for aperture efficiency:

$$\eta_s = \exp[-(4\pi\sigma/\lambda)^2]$$

For σ/λ of 1/78 or 1 mil tolerance, the antenna gain would degrade by 13 %. An absolute gain measurement of the antenna (to be done in the future) would check this number.

5 Beam mapping

Figure 9 is the sketch of the beam-mapping set-up. The *D*-band (149.7 GHz) source, identical to the noise source used in the MAT experiment [5] is weak with a maximum transmitted power of 0.7 mW (–1.5 dBm). Hence, the source was modulated at 1 kHz and a reference signal sent to a lock-in amplifier for DC detection.

Results: Only near-field measurements were done. Figure 10a gives the measured horn profile alone; Figure 10b is a plot from *DADRA* as a comparison. Figure 11 gives the measured beam cross-polarization at 22 inches away. Figure 12 gives co-polarization at 15 feet away. As a comparison, the near-field plot from *DADRA* is given in Figure 13. The FWHM of the measure plots agree with *DADRA* up to 10 % -- 2 degree for the horn profile, 0.05 degree for beam pattern. The beam first side-lobe occurs at about -6dB below maximum, also in agreement with *DADRA*.

The angles from the measured plot are relative. Negatives value points toward the floor. This could explain the shoulder on the left half of Figure 12. A plausible resolution is to have the source located at a higher plane, pointing down towards the antenna for measurements.

6 Conclusions

A more elaborate set-up for beam mapping that avoids reflection from the ground is needed to map the far-field beam pattern. Antenna tolerance could be tested by doing absolute gain measurements.

7 Acknowledgements

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Appendix A

Cassegrain System Parameters

Conic section polar equation	$r = \dfrac{ep}{1 - e\cos\theta}$
Primary focal ratio	f/D
Equivalent focal length	$F = Mf$
Equivalent focal ratio	F/D
Secondary interfocal length	$f_s = 2c = f + z = \dfrac{d(M+1)(16f^2M - D^2)}{16MDf} = \dfrac{2pe^2}{1-e^2}$
Exit focal ratio	f_s/d
Secondary eccentricity	$e = \dfrac{M+1}{M-1}$
Secondary directrix	$p = \dfrac{e^2 - 1}{e^2}c$
Half - angle subtended by primary	$\theta_p = 2\tan^{-1}(D/4f)$
Half - angle subtended by secondary	$\theta_s = 2\tan^{-1}(D/4F)$

Appendix B

Optics Definitions and Conventions

(adapted from L. Page : Convention for *Map* Optics Calculations, February 4, 1998)

Beam Patterns and Gain

Normalized antenna response to power is

$$P_n(\theta,\phi)=\frac{|\psi(\theta,\phi)|^2}{|\psi|^2_{\max}}$$

where ψ is the scalar electric field in units (power)$^{1/2}$/*distance* and is evaluated at a fixed distance from the source.

Full-width at half maximum (*FWHM)* of a symmetric beam is the angle at which

$$P_n(\theta_{FWHM}/2)=1/2 \text{ (or -3 dB)}$$

At the output of an antenna system, one measures the power given by

$$W=\frac{1}{2}\int_{\Omega}\int_{\nu}A_e(\nu)S_\nu(\theta,\phi)P_n(\theta,\phi)d\Omega d\nu \qquad \text{(Watts)}$$

where A_e is the effective area of the antenna and $S_\nu(\theta,\phi)$ is the brightness of the sky.

The *directivity* is

$$D_{\max}=\frac{|\psi|^2_{\max}}{|\psi_{avg}|^2}=4\pi\frac{|\psi|^2_{\max}}{\int_{\Omega}|\psi(\theta,\phi)|^2 d\Omega}=\frac{4\pi}{\int P_n(\theta,\phi)d\Omega}=\frac{4\pi}{\Omega_A}=\frac{4\pi A_e}{\lambda^2}$$

where Ω_A is the total solid angle of the normalized antenna pattern, and $|\psi_{avg}|^2$ is the total power averaged over the sphere.

When there is no ohmic losses in the telescope, we write the *gain*

$$g(\theta,\phi)=D_{\max}P_n(\theta,\phi)$$

The maximum gain, g_m, is just the *directivity.* In the text, this maximum gain is loosely called " the gain".[11]

Feed as Source of Radiation

Consider a feed of maximum gain g_m with total power W. If one were to measure the flux (power/area) at the maximum, one finds

$$I=\frac{g_m W}{4\pi r^2} \quad (\text{W/m}^2)$$

The gain is obtained by measuring the field at a distance r from the feed

$$g(\theta,\phi)=\frac{4\pi r^2|\psi(r,\theta,\phi)|^2}{\text{Total power from the feed}}$$

[11] Short for "the gain above isotropic". For an isotropic emitter, "the gain" is unity (0 dB).

The gain is important because it is the quantity that indicates the antenna's immunity to off-axis sources.

The gain is always normalized so that

$$\int \frac{g(\theta,\phi)}{4\pi} d\Omega = \int_{S'} \frac{|\psi(x',y')|^2}{\text{Total feed power}} dx'\,dy' = 1$$

where the prime coordinates are for the aperture of the mirror.

Edge Taper, y_e

The edge taper is usually given in dB.

$$y_e = 10\log\left(\frac{I_{\text{edge}}}{I_{\max}}\right)$$

where I is the intensity of the beam (or current density on the mirror). For a symmetric beam, $I_{\max}$ is also I_{center} .

Appendix C

Brief on *DADRA* output and guides to reading Figures 2, 3, 7 and 13.

DADRA gives four columns of unnormalized complex *E*-field --two columns (one the real part, the other the imaginary part) of co-polarization field, and the other two for cross-polarization field. Without loss of generality, I chose right-handed circularly polarized *E*-field as the co-polar field.

I then uses *IDL* routine acontr.pro written by L. Page for *MAP* optics analysis (with minor modifications and renamed acontr_circ.pro) to give antenna beam contours and gain plots (Figure 2, 3, 5 and 6). Each figure contains four plots. The top two are beam contour plots (one for co-polar, the other cross-polar) and directly below them are their respective antenna gain plots.

Beam contours are just of *E*-field magnitudes (in spherical coordinate) projected on a grided-plane. The axis of the antenna defines the *z*-axis. So, at a fix far-field distance *r*, the *E*-field from an antenna with azimutal symmetry varies only with the polar angle θ. The grided plane has axis θ_x and θ_y (shown as XP and YP in contours of Figures) where $\theta^2 = \theta_x^2 + \theta_y^2$.

Antenna gain in dB, calculated from definitions given in Appendix B, is plotted against the same polar angle θ directly below the contour plots.

DADRA also gives the current distributions on the two mirrors. Using *IDL* routine curd.pro by L. Page (again with minor modifications), the current densities of the mirrors are projected onto a 2D plane perpendicular to the antenna axis. Figure 4 shows four current plots. The right column gives currents from the primary mirror; the left gives those from the secondary. We only care about the bottom two plots – those of the total current densities, J_T. (Note: $J_T^2 = J_X^2 + J_Y^2 + J_Z^2$) The absolute values of the current densities are not important. We use their relative values to compute the edge taper on the dishes.